$$z_n = -\frac{x\,\epsilon_n\,\eta^{2n}}{2n\,(2n-1)} - \epsilon_{0n}\,x \tag{13}$$

These results may be extended to nondelta planforms in the same way as described by Jones.[2] However, it is always useful to remember the drawbacks of any analysis when several idealizations have been made to arrive at simple solutions. Thus the slenderness assumption violates the Kutta condition in subsonic flow by ignoring the presence of the trailing edge. As a consequence, the lift near the trailing edge is considerably overestimated. An emperical correction suggested by Laidlaw and Hsu[7] when used with the slender wing results was found to be very useful and is therefore repeated here

$$\Delta C_{p_c}\,(x,\ \eta) = \sqrt{1-\left(\frac{x}{c}\right)^2}\ \Delta C_{p_i} \tag{14}$$

where ΔC_{pc} is the corrected value, ΔC_{pi} is from slender wing theory, and c is the local chord. The result is valid for steady and unsteady cases and has been successfully used in flutter analysis. In supersonic flow, the presence of the trailing edge is felt only through the boundary layer and for most purposes it is sufficient to assume that pressure equalization takes place through an oblique shock. The slender wing solution is therefore valid quite close to the trailing edge. The slender theory, however, fails to predict the wave drag.

As a matter of interest, the spanwise pressure and ordinate distributions are shown in Fig. 2 for $n = 1$–4. It is seen that these wings have a leading edge droop which helps the flow to turn smoothly, and hence, the predicted aerodynamic properties may be expected to be close to reality.

The analytical results of this note augment the solutions of Ref. 2 for the flat delta, and of Ref. 3 for the delta with a linear twist distribution. The simplicity of the results along with the Laidlaw-Hsu correction should make them useful where quick estimates of the pressure distribution are required.

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Influence of Static Aeroelasticity on Oblique Winged Aircraft

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Introduction

BENDING deformation has an important influence on the static aeroelastic behavior of slender wings of moderate sweep. Bending deformation is responsible for the strong tendency of sweptforward wings to diverge and accounts for the fact that moderately sweptback wings will not diverge. Spanwise aerodynamic load redistribution is caused, in part, by bending deformation and accounts for the poor aileron reversal characteristics of sweptback wings. Interest has recently been expressed about the impact of static aeroelasticity on the lateral control of oblique winged aircraft.[1,2] The purpose of this Note is to illustrate, by use of a simple example, together with results from published literature, an aeroelastic phenomenon which the author believes to be unique to oblique winged aircraft. This phenomenon is the aeroelastic roll moment and occurs because upward bending deflection of the sweptforward wing generates additional lift while the converse is true of sweptback wings.

Discussion

For problems involving swept wing static aeroelasticity, bending and torsional deformations are coupled. Excellent discussions of the formulation of such problems may be found, for instance, in Refs. 3–5. The essential features of the interaction between wing deformation, aerodynamic forces and aileron trimming can be illustrated by means of a simple example. Consider the constant chord, slender, oblique wing shown in Fig. 1. This elastically symmetrical wing model is swept at an angle Λ to the airstream and has full-span ailerons for lateral control. Pitch, yaw, and plunge degrees of freedom are not permitted, but, as will be shown, the ailerons must be deflected to guarantee static equilbrium in roll.

To simplify the aeroelastic analysis, assume that the wing is uncambered and that the straight elastic axis and the line of aerodynamic centers coincide. In this case, only

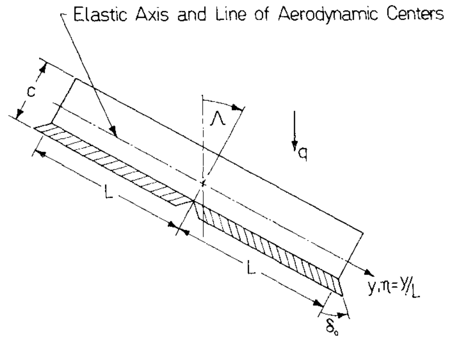

Fig. 1. Oblique wing idealization.

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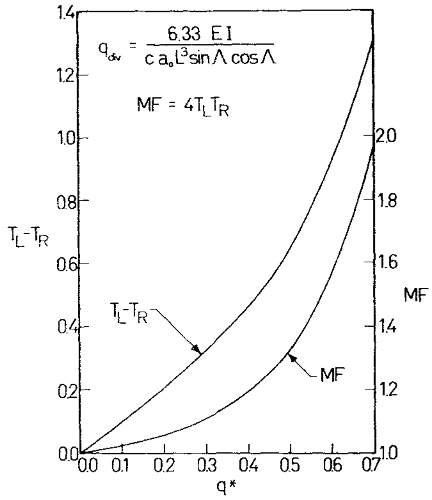

Fig. 2 Aeroelastic roll moment factor ($T_L - T_R$) and bending moment magnification factor (MF) vs q^* (q/q_{Div}).

bending deformation is important. Further assume that the wing is sufficiently slender so that elementary beam bending theory may be used. If chordwise cross sections are used, the governing nondimensional differential equation for a wing with aerodynamic strip theory loads can be written as (cf. Ref. 5, pp. 479–481)

$$\frac{d^4w}{d\eta^4} + \lambda \frac{dw}{d\eta} = \frac{p_0 L^3}{EI} + \beta\delta(\eta) \qquad -1 \leq \eta \leq 1 \tag{1}$$

In Eq. 1, the following terminology is used:

a_o = two-dimensional sectional lift-curve slope
p_o = applied load per unit length, a constant
w = bending deformation, nondimensionalized with respect to L
β = $qcc_l\, L^3 \cos^2\Lambda/EI$
λ = $qca_oL^3 \sin\Lambda\cos\Lambda/EI$

Assume the ailerons to be deflected asymmetrically such that

$$\delta(\eta) = \begin{cases} \delta_0 & 0 < \eta \leq 1 \\ -\delta_0 & -1 \leq \eta < 0 \end{cases} \tag{2}$$

δ_o is a constant which is determined from the following roll moment static equilibrium condition.

$$\text{Rolling Moment} \sim \int_{-1}^{1} \delta(\eta)\eta d\eta + \frac{a_0 \tan\Lambda}{c_{l\delta}} \int_{-1}^{1}\left(-\frac{dw}{d\eta}\right)\eta d\eta = 0 \tag{3}$$

Integration yields

$$\delta_0 = \left(\frac{a_0}{c_{l\delta}}\right) \tan\Lambda \int_{-1}^{1} \frac{dw}{d\eta}\eta d\eta \tag{4}$$

The presence of the aeroelastic roll moment tendency is clearly seen in Eq. 4. For a flat rigid wing, δ_o would be zero. Equation 1 may be divided into two regions for ease of solution. Equivalent boundary conditions for the entire wing in static equilibrium are found if it is assumed that each wing half is clamped at the root and free of shear and bending moment at the wing tips. In this case, a closed form solution to the problem may be found.

The assumption that both wing portions are clamped at the root assumes that a fuselage restraint is present at the center of the span shown in Fig. 1. For a freely flying wing, the validity of such an assumption is subject to question. The present study assumes the presence of a restraint such as would be applied by a conventional fuselage.

Reference 5 (pp. 311–314) details such a solution to Eq. 1 for both sweptforward and sweptback wings. Since the slope of the deformed elastic axis is the quantity which defines the aeroelastic lift forces, it is convenient to adopt the notation that $\Gamma(\eta) = dw/d\eta$. It may be verified that, in the region $-1 \leq \eta \leq 0, \Gamma(\eta)$ is given by

$$\Gamma_L(\eta) = \frac{1}{a^3}\left[\frac{p_0 L^3}{EI} - \beta\delta_0\right] \left[1 - \frac{e^{-a(1+\eta)} + 2e^{a(1+\eta)/2}\cos f(1+\eta)}{e^{-a} + 2e^{a/2}\cos f}\right] \tag{5}$$

where $a = \lambda^{1/3}$ and $f = (3)^{1/2}/2a$. In the region $0 \leq \eta \leq 1$, $\Gamma(\eta)$ is given by

$$\Gamma_R(\eta) = \frac{1}{a^3}\left[\frac{p_0 L^3}{EI} + \beta\delta_0\right] \left[1 - \frac{e^{a(1-\eta)} + 2e^{-a(1-\eta)/2}\cos f(1-\eta)}{e^{a} + 2e^{-a/2}\cos f}\right] \tag{6}$$

With these solutions, δ_o is found, from Eq. 4, to be

$$\delta_0 = \frac{p_0 L^3}{\beta EI}\left(\frac{T_L - T_R}{T_L + T_R}\right) \tag{7a}$$

where

$$T_L = \frac{e^{-3a/2} - \cos(3/2)^{1/2}a + (3/2)^{1/2}\sin(3/2)^{1/2}a}{a^2(e^{-3a/2} + 2\cos(3/2)^{1/2}a)} \tag{7b}$$

$$T_R = \frac{e^{3a/2} - \cos(3/2)^{1/2}a - (3)^{1/2}\sin(3/2)^{1/2}a}{a^2(e^{3a/2} + 2\cos(3/2)^{1/2}a)} \tag{7c}$$

The numerator in Eq. (7a) is proportional to the wing aeroelastic roll moment in the absence of aileron action. The denominator occurs because aileron deflection introduces further wing deformation. The factor $T_L - T_R$ is plotted vs the variable q^* in Fig. 2. q^* is the ratio q/q_{Div} where q_{Div} is the divergence q of a clamped, uniform-property sweptforward wing with elastic axis length L. For this value of λ, since the denominator in Eq. (7b) is identically zero, T_L becomes infinite. From Fig. 2, it is seen that the roll moment increases rapidly as q^* increases.

With the antisymmetrical application of ailerons, lateral static equilibrium is effected. Figure 3 shows a graph of the parameter $\gamma = \beta\delta_o/(p_oL^3/EI)$.

Although it cannot be determined from this figure, at q^* equal to unity, γ is also equal to unity. This occurs because the bending deformation of the sweptforward wing, caused by aileron deflection, reinforces the aileron. As q^* increases, the sweptforward aileron furnishes powerful control. From Fig. 3 it is seen that γ (and thus δ_o) tends to infinity as q^* tends to the value 4.335. This q^* value is equivalent to a value of λ equal to 27.455. This behavior

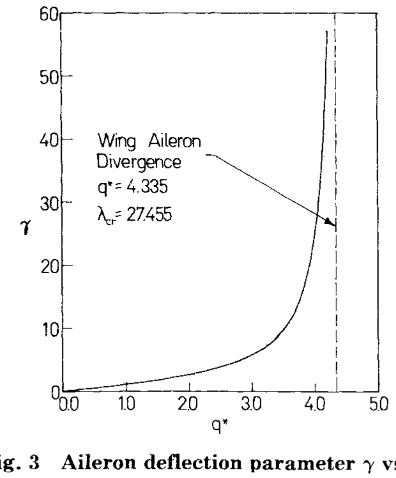

Fig. 3 Aileron deflection parameter γ vs q^*.

occurs because the term T_L becomes equal to $-T_R$ at this point.

Although the physical assumptions of the problem are no longer valid at this large value of λ, the result deserves additional discussion. For this value of λ (termed λ_{cr}) the deformation of the wing is unbounded because γ (and thus δ_o) is infinite. This behavior is, in the classical sense, a static divergence instability and is referred to in Ref. 2 as "wing-aileron divergence." In that reference, the authors studied the problem with a small perturbation approach and utilized the Galerkin method to obtain the approximate value $\lambda_{cr} = 32$. The present result is seen to be an exact determination of this hypothetical instability.

It should be noted that the addition of the trimming device precludes the discovery, from these equations, of bending divergence of the sweptforward wing. The reader is cautioned, however, that only static equilibrium is being investigated here. An additional, separate analysis is necessary to study aeroelastic stability.

The antisymmetrical application of ailerons assures lateral static equilibrium and also causes a different bending moment distribution on each wing. However, at the wing centerline or root, the bending moment is continuous because of the roll equilibrium requirement. The bending moment at the wing centerline, about an axis perpendicular to the wing elastic axis, is found to be

$$M_r = (2p_0L^2)T_RT_L \tag{8}$$

Without aeroelastic effects the bending moment at the wing root, due to the uniform load p_o, would be $M_{rigid} = p_oL^2/2$. The magnification of the bending moment due to aeroelasticity may be defined as

$$M.F. = M_r/M_{\text{rigid}} = 4T_RT_L \tag{9}$$

Figure 2 shows the quantity $4T_RT_L$ plotted vs q^*. M.F. is found to be 1.332 at a value of q^* to one-half. M.F. rises dramatically thereafter.

Conclusions

It is important to draw attention to the idealizations, and therefore shortcomings, present in this study. First of all, in real aircraft, bending *and* torsion flexibility are present and are elastically coupled. More importantly, the lift (here represented by p_o) is not uniformly distributed over the wing span, nor is the span of constant chord. The presence of three-dimensional aerodynamic effects due to finite wing-body combination and a tapered, more flexible wing will most certainly modify the simplistic formulae presented here.

The significance of these results lies in the fact that they present, via a closed-form solution, the essential features of the aeroelastically induced roll moment on the oblique wing. The results show that, depending on the magnitude of the ratio $q^* = q/q_{Div}$, antisymmetrical aileron trim is necessary for lateral static equilibrium of an elastically symmetric wing. The wing root bending moment is adversely affected as q^* increases. Since p_o is proportional to a large fraction of the aircraft gross weight (this fraction is nearly equal to unity, even after aeroelastic effects on total lift are considered) one may also speculate on the additional demands upon control surfaces as q^* increases. Although more sophisticated analysis is in order, this phenomenon appears to be an additional design consideration for an oblique winged aircraft.

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Hypersonic Flow Over Blunted Slender Wedges

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THE problem of inviscid hypersonic flow over blunted wedges has received the attention of many investigators. Theoretically, two well-known methods of calculation are due to Cheng[1] and Chernyi.[2] An interesting feature of Cheng's analysis is the oscillatory approach in pressure and shock waveshape for long downstream distances. Earlier results of Chernyi did not exhibit any oscillatory behavior and its absence was suspected to be due to the inclusion of the kinetic energy term in the analysis.[3] However, it has been reported recently by Schnieder[4] that Chernyi's approach also predicts an oscillatory decay but the wavelength and amplitude of the oscillation are quite different. An alternative approach which gives a nonoscillatory decay was suggested by Stollery[5] for the case of viscous interaction on a concave surface. The method was later extended to the case of thin blunted wedges by Stollery et al.[6] They used the tangent wedge approximation, which gives the correct downstream limit, in place of the Newton-Busemann pressure law employed in Cheng's theory, and integrated the resulting equation numerically. The modified method suppressed oscillatory behavior and the results agreed with the method of characteristics downstream.

The purpose of this Note is to show that the results of Ref. 6 can be obtained in closed form for the pressure and shock waveshape by integrating the equations analytically.[7] Further, it is indicated that the estimates of the method near the leading edge can be improved by incorporating modifications suggested by Kemp[8] to account for the effects of γ (ratio of specific heats).

Analysis

The blunt leading edge introduces a strong entropy gradient in the downstream flow which significantly modifies the pressure and heat transfer on the afterbody. By an analysis of the flow within the entropy layer, Cheng obtains the pressure-area relation as

$$p_e(y_e - y_b)^\gamma = \text{constant} \tag{1}$$

Here y_b is the body shape, y_e the edge of the entropy layer, p_e the pressure (assumed constant across the entro-

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